

Example 1. Multiply $x^3 + 2x^2 - x - 4$ by $2x^2 - x + 4$

Solution:

$$\begin{array}{r} x^3 + 2x^2 - x - 4 \\ 2x^2 - x + 4 \\ \hline 2x^5 + 4x^4 - 2x^3 - 8x^2 \\ - \quad x^4 - 2x^3 + x^2 + 4x \\ \hline 4x^3 + 8x^2 - 4x - 16 \\ 2x^5 + 3x^4 \quad + x^2 \quad - 16 \end{array}$$

Example 2. Multiply $b^2 - 2ab + a^2$ by $a^2 + b^2 + ab$

Solution: We first arrange the expressions in descending powers of a .

$$\begin{array}{r} a^2 - 2ab + b^2 \\ a^2 + ab + b^2 \\ \hline a^4 - 2a^3b + a^2b^2 \\ a^3b - 2a^2b^2 + ab^3 \\ a^2b^2 - 2ab^3 + b^4 \\ \hline a^4 - a^3b \quad - ab^3 + b^4 \end{array}$$

Check.

Note that the equality $(a^2 - 2ab + b^2)(a^2 + ab + b^2) = a^4 - a^3b - ab^3 + b^4$ should be an identity. Since an identity is a statement of equality that is true for all values of the variables, we can obtain a helpful check of this equality by substituting numbers for the letters.

Thus, if we let $a = 2$ and $b = 3$,

then $a^2 - 2ab + b^2 = 4 - 12 + 9 = 1$
 $a^2 + ab + b^2 = 4 + 6 + 9 = 19$
 so that $(a^2 - 2ab + b^2)(a^2 + ab + b^2) = 1 \times 19 = 19$
 But $a^4 - a^3b - ab^3 + b^4 = 16 - 24 - 54 + 81 = 19$

This shows that the equality

$$(a^2 - 2ab + b^2)(a^2 + ab + b^2) = a^4 - a^3b - ab^3 + b^4$$

is true when $a = 2$ and $b = 3$. By a similar procedure we could show it to be true for other pairs of values for a and b . It is reasonable to conclude that the multiplication is correct.

The work may also be checked by simply doing it again.

Exercises ^[A-1]

Multiply:

- | | |
|-------------------------------|---------------------------------|
| 1. $x^2 + 2x + 3$ by $x + 5$ | 5. $x^2 + 2xy + y^2$ by $x - y$ |
| 2. $3a + 7$ by $2a + 1$ | 6. $a^2 - 4a + 5$ by $a - 1$ |
| 3. $2a^2 - 5a + 4$ by $a + 3$ | 7. $4r^2 - 2r + 1$ by $3r - 4$ |
| 4. $x + 2x^2 - 5$ by $3x - 2$ | 8. $6a - 5 + a^2$ by $a + 2$ |

9. $x^2 + xy + y^2$ by $x - y$
 10. $x^3 + x^2 + x + 1$ by $x - 1$
 11. $p^3 - 2p - 1$ by $2p + 1$
 12. $2x^2 + 3x^3 - x$ by $1 + 2x$
 13. $y^2 - 2x^2y - x^4$ by $3x^2 - y$
 14. $x + 2y + 3z$ by $2x - y - 2z$
 15. $2a^2 + b^2 + 2ab$ by $2a^2 + b^2 - 2ab$
 16. $4c^2 - 3d^2 + cd$ by $2d^2 + 5cd + c^2$

17. The factors of an expression are $a^2 - 2ab + 2b^2$ and $a - b$. What is the expression?

Multiply as indicated:

18. $(3x - 5)(3x - 5)$
 19. $(p - 2q - 1)(p - 2q - 1)$
 20. $(3x + y + 1)(3x - y + 1)$
 21. $(2a - 7)(2a - 7)(2a - 7)$
 22. $x(x + z)(x + z)$
 23. $(x + 1)(x - 3)(x - 3)$
 24. Add the product $(x - y)(x - y)$ to the product $(2x + y)(2x + y)$.
 25. Subtract the product $(a - b + 2c)(a - b + 2c)$ from the product $(2a + b)(2a + b)$.

Exercises [A-2]

Multiply:

1. $2y - 3$ by $y + 4$
 2. $y - 3z$ by $-y + 3z$
 3. $3b^2 + b - 6$ by $b - 1$
 4. $2z^2 - 5z + 2$ by $2z - 5$
 5. $2a^2 + ab - b^2$ by $2a - b$
 6. $3x^2 + 4xy - y^2$ by $2x - 3y$
 7. $7 + 4n^2 - 2n$ by $3n - 5$
 8. $4ab - a^2 + 3b^2$ by $a - b$
 9. $x^2 - xy + y^2$ by $x + y$
 10. $2a^2 + a^3 + 1 - a$ by $a + 3$
 11. $y^4 + 2y^2 + 1$ by $3y + 4$
 12. $p^3 + 3p^2q - pq^2$ by $3p + q$
 13. $x^2 - 2x + 1$ by $x^2 + 2x + 1$
 14. $3a^2 + 2ab + b^2$ by $a^2 - ab - b^2$
 15. $3x^2 - x^3 - 2 - x$ by $2x + x^2 - 2$
 16. $a^3 - a^2b + 4ab^2$ by $2b^2 + 2ab - a^2$

17. Find the expression whose factors are $c^2 - cd + d^2$ and $c + d$.

Multiply as indicated:

18. $(a + b + c)(a + b + c)$
 19. $(2x + y - z)(2x + y - z)$
 20. $(x + 2)(x + 2)(x + 2)$
 21. $3(2p + 3)(2p + 3)$
 22. $(a - 2b)(a + 2b)(a - 2b)$
 23. $x(x + 4)(x + 4)$
 24. Add the product $(2m - 3n)(2m - 3n)$ to the product $(n - m)(n - m)$.
 25. Subtract the product $(3x - y)(3x - y)$ from the product $(x + y - 2)(x + y - 2)$.

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1. $x^3 + 7x^2 + 13x + 15$

2. $6a^2 + 17a + 7$

3. $2a^3 + a^2 - 11a + 12$

4. $6x^3 - x^2 - 17x + 10$

5. $x^3 + x^2y - xy^2 - y^3$

6. $a^3 - 5a^2 + 9a - 5$

7. $12r^3 - 22r^2 + 11r - 4$

8. $a^3 + 8a^2 + 7a - 10$

9. $x^3 - y^3$

10. $x^4 - 1$

11. $2p^4 + p^3 - 4p^2 - 4p - 1$

12. $6x^4 + 7x^3 - x$

25. $3a^2 + 6ab - 4ac + 4bc - 4c^2$

13. $-3x^6 - 5x^4y + 5x^2y^2 - y^3$

14. $2x^2 + 3xy + 4xz - 7yz - 2y^2 - 6z^2$

15. $4a^4 + b^4$

16. $4c^4 + 21c^3d + 10c^2d^2 - 13cd^3 - 6d^4$

17. $a^3 - 3a^2b + 4ab^2 - 2b^3$

18. $9x^2 - 30x + 25$

19. $p^2 - 4pq - 2p + 4q^2 + 4q + 1$

20. $9x^2 + 6x - y^2 + 1$

21. $8a^3 - 84a^2 + 294a - 343$

22. $x^3 + 2x^2z + xz^2$

23. $x^3 - 5x^2 + 3x + 9$

24. $5x^2 + 2xy + 2y^2$

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1. $2y^2 + 5y - 12$

2. $-y^2 + 6yz - 9z^2$